Verification Techniques for Smart Contracts in Agda

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What are smart contracts?
Smart contracts are transactions that are defined through software and executed automatically when conditions in the blockchains are met.

Smart contracts in the cryptocurrency Bitcoin are written in the language Script.
EVM vs Script

- Ethereum Virtual Machine and Bitcoin Script are similar.
- **EVM [5]:**
  - EVM extends and modifies Bitcoin Script, especially it
    - adds loops (jumps),
    - allows calls to other contracts,
    - adds cost of execution of instructions (gas) to guarantee termination.
- **Bitcoin Script [5]:**
  - without loops.
  - without possibility to calling other contracts.
The scripting language for Bitcoin is stack-based, and similar to Forth. The script in Bitcoin has a set of commands called Operation Codes such as OP_HASH160, OP_ADD, OP_EQUAL and OP_VERIFY. Several standards scripts [3] are used in Bitcoin such as the Pay-to-Public-Key-Hash (P2PKH) and Multi-signature (P2MS) scripts.
Contribution

- Operational semantics.
- Verification of example bitcoin scripts.
- An Agda library supporting it.
Several opcodes have been introduced and formalised in Agda [1, 2] such as **OP_EQUAL**, **OP_IF**, **OP_ELSE**, and **OP_ENDIF**...

**Example of **OP_EQUAL**:

\[
< 2 > < 3 > \text{OP_ADD} < 5 > \text{OP_EQUAL}
\]

- The stack evolves as follows:

\[
[] \\
[< 2 >] \\
[< 2 >, < 3 >] \\
[< 5 >] \\
[< 5 >, < 5 >] \\
[1]
\]
Example of **OP_IF**, **OP_ELSE**, and **OP_ENDIF**:

```
OP_IF <Alice’s PubKey> OP_CHECKSIG
OP_ELSE <Bob’s PubKey> OP_CHECKSIG OP_ENDIF
```

- assume stack contains <sig> <1> (1 = True)
  The script will succeed if sig is a signature for the transaction using Alice’s private key.
- In case it contains <sig> <0> signature need to be for Bob’s Pub Key.
All Bitcoin scripts consist of a locking script (provided by a person who sends coins) and an unlocking script (provided by someone who wants to access it).

- **P2PKH** has a locking script (scriptPubKey) and an unlocking script (scriptSig) [4].

  scriptPubKey:  \( \text{OP\_DUP \ OP\_HASH160 \ <pubKeyHash> \ OP\_EQUAL \ OP\_VERIFY \ OP\_CHECKSIG} \)

  scriptSig:  \( <\text{sig}> \ \text{<pubKey>} \)

  - success if running first scriptSig and then scriptPubKey succeeds with not false on top of stack.
The operational semantics of opcodes depends on \( \text{Time} \times \text{Msg} \times \text{Stack} \). We define it in Agda as the record type `StackState`.

- All opcodes is given as `InstructionBasic`.
  - Opcodes can fail, for example if there are not enough elements on the stack as required by the operation.

The operational semantics of \( p : \text{InstructionBasic} \)

\[
[ p ]s : \text{StackState} \rightarrow \text{Maybe StackState}
\]

- As an example, the semantics of \([ \text{opEqual} ]s\):

\[
[ \text{opEqual} ]s < t, \text{msg}, s > = \text{nothing} \quad \text{if s has height} \leq 1 \\
[ \text{opEqual} ]s < t, \text{msg}, s0 :: s1 :: s > = \text{just} < t, \text{msg}, i :: s >
\]

where \( i = 1 \) if \( s0 = s1 \) and \( i = 0 \) otherwise.
Time: there are instructions for checking that a certain amount of time has passed, and time is used for checking against the current time.

opCHECKLOCKTIMEVERIFY: allows to lock a resource until a certain amount of time has passed.

Msg is the part of the transaction to be signed when a signature is required.
Hoare triple and weakest precondition

We define for $\Phi, \Psi \subseteq \text{State}$ and $p$ a Bitcoin Script the Hoare triple with weakest pre condition

For the unlocking script of P2PKH we show:

Therefore in order to unlock one needs to provide a script which computes the pubkey hashing to the pbkhh and a corresponding signature.
Hoare triple and weakest precondition

We define for $\Phi, \Psi \subseteq \text{State}$ and $p$ a Bitcoin Script the Hoare triple with weakest pre condition

$$\langle \Phi \rangle \leftrightarrow p \langle \Psi \rangle : \iff$$

$$(\forall s \in \text{State1.} \Phi(s) \rightarrow \Psi([p]s))$$

$$(\forall s \in \text{State1.} \Psi([p]s) \rightarrow \Phi(s))$$

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Hoare triple and weakest precondition

We define for $\Phi, \Psi \subseteq \text{State}$ and $p$ a Bitcoin Script the Hoare triple with weakest precondition

\[
\langle \Phi \rangle \leftrightarrow p \langle \Psi \rangle \iff \\
(\forall s \in \text{State1.} \Phi(s) \rightarrow \Psi([p]s)) \\
\land (\forall s \in \text{State1.} \Psi([p]s) \rightarrow \Phi(s))
\]

For the unlocking script of P2PKH we show:

\[
(\langle \Phi \rangle \leftrightarrow \text{scriptSig} \langle \text{accept} \rangle) \\
\iff \text{the two top elements of the stack consist of a pubkey hashing to the pbkh and a corresponding signature.}
\]

Therefore in order to unlock one needs to provide a script which computes the pubkey hashing to the pbkh and a corresponding signature.
Our library

- Develop a library in Agda and prove correctness of smart contracts [1, 2].

We prove the theorem for the Hoare triple for $\text{prog1} ++ (\text{prog2} ++ \text{prog3})$ is given as follows:

```agda
theorem : < precondition >iff prog1 ++ (prog2 ++ prog3) < postcondition >
theorem = precondition <<< prog1 > proof1 >>>
intermediateCond1 <<< prog2 > proof2 >>>
intermediateCond2 <<= proof3 >
intermediateCond3 <<< prog3 > proof4 >>> e postcondition p
```

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Proof of Correctness of the P2PKH script using the Step-by-Step approach

- **P2PKH script:**

  \[
  \text{scriptP2PKH}^b : (\text{pbkh} : \mathbb{N}) \rightarrow \text{BitcoinScriptBasic}
  \]

  \[
  \text{scriptP2PKH}^b \text{ pbkh} = \text{opDup} :: \text{opHash} :: (\text{opPush} \text{ pbkh}) :: \text{opEqual} :: \text{opVerify} :: [ \text{opCheckSig} ]
  \]

- **Intermediate conditions** \(\text{accept}_1, \text{accept}_2, \text{etc}\)
  - For example:
    - \(\text{accept}_1^s \ m \ t \ st \iff \exists \ pbk, \ sig, \ st'. st \equiv pbk :: sig :: st'
      \land \text{IsSigned} m \ sig \ pbk\)
    - \(\text{accept}_2^s \ m \ t \ st \iff \exists x, \ pbk, \ sig, \ st'. st \equiv x :: pbk :: sig :: st'
      \land x > 0 \land \text{IsSigned} m \ sig \ pbk\)

- **Proofs** \(\text{correct-1}, \text{correct-2}, \text{etc}\)
  - \(\text{correct-1} : < \text{accept}_1 > \iff([ \text{opCheckSig} ]) < \text{acceptState} >\)
  - \(\text{correct-2} : < \text{accept}_2 > \iff([ \text{opVerify} ]) < \text{accept}_1 >\)
Weakest precondition

\[ w_{\text{PreCondP2PKH}} : (pbkh : \mathbb{N}) \rightarrow \text{StackPredicate} \]

\[ w_{\text{PreCondP2PKH}}\ pbkh\ time\ m\ [] = \perp \]

\[ w_{\text{PreCondP2PKH}}\ pbkh\ time\ m\ (\times :: []) = \perp \]

\[ w_{\text{PreCondP2PKH}}\ pbkh\ time\ m\ (\text{pubKey} :: \text{sig} :: \text{st}) = \]

\[ (\text{hashFun} \ \text{pubKey} \equiv pbkh) \land \text{IsSigned} \ m \ \text{sig} \ \text{pubKey} \]

Prove the weakest precondition for the P2PKH script as follows:

\[ \text{theoremP2PKH} : (pbkh : \mathbb{N}) \rightarrow < w_{\text{PreCondP2PKH}} pbkh > \Leftrightarrow \text{scriptP2PKH}^b \ pbkh \ < \ \text{acceptState} > \]

\[ \text{theoremP2PKH} \ pbkh = w_{\text{PreCondP2PKH}} pbkh \triangleleft \triangleleft ( [\text{opDup}] \langle \text{correct-6 pbkh} \rangle \]

\[ \text{accept}_5 \ pbkh \triangleleft \triangleleft ( [\text{opHash}] \langle \text{correct-5 pbkh} \rangle \]

\[ \text{accept}_4 \ pbkh \triangleleft \triangleleft ( [\text{opPush pbkh}] \langle \text{correct-4 pbkh} \rangle \]

\[ \text{accept}_3 \ \triangleleft \triangleleft ( [\text{opEqual}] \langle \text{correct-3} \rangle \]

\[ \text{accept}_2 \ \triangleleft \triangleleft ( [\text{opVerify}] \langle \text{correct-2} \rangle \]

\[ \text{accept}_1 \ \triangleleft \triangleleft ( [\text{opCheckSig}] \langle \text{correct-1} \rangle \text{e acceptState} \triangleleft \text{p} \]

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Conclusion

- Differences between precondition and weakest precondition.
- Implemented theorems for verifying Bitcoin script using conditions.
- Our goal is to develop our approach into a framework for developing smart contracts that are correct by construction.
- Applied our approaches to P2PKH and P2MS.
Thank you for listening.
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