Forwarders as Process Compatibility

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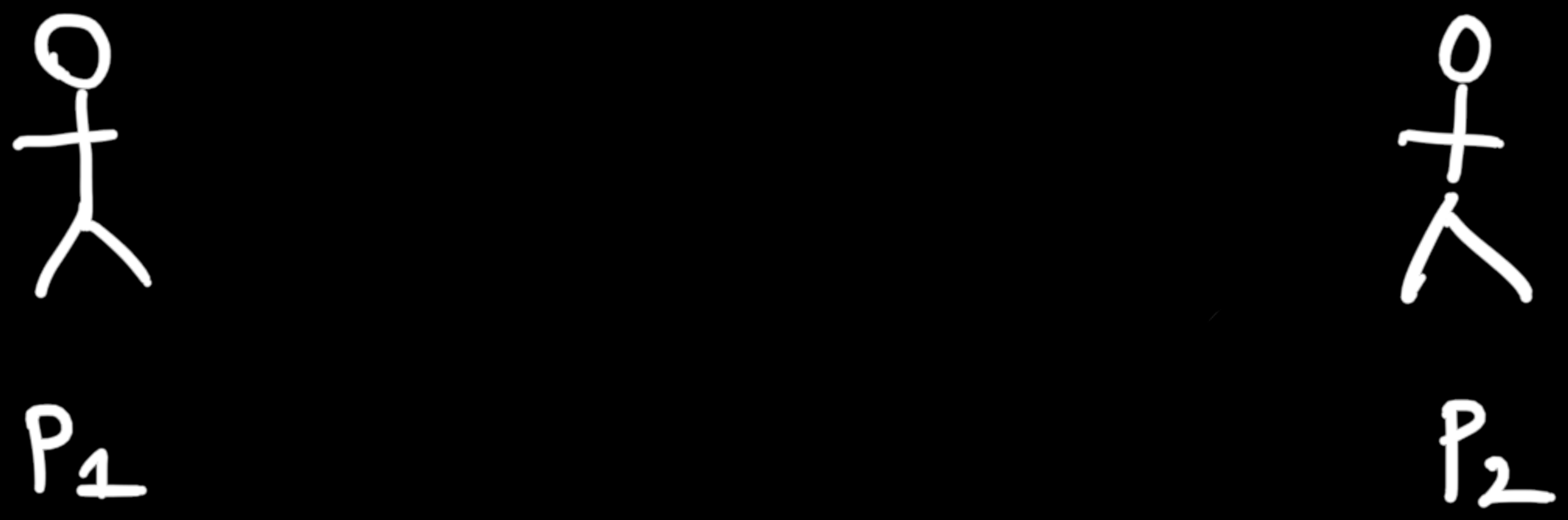
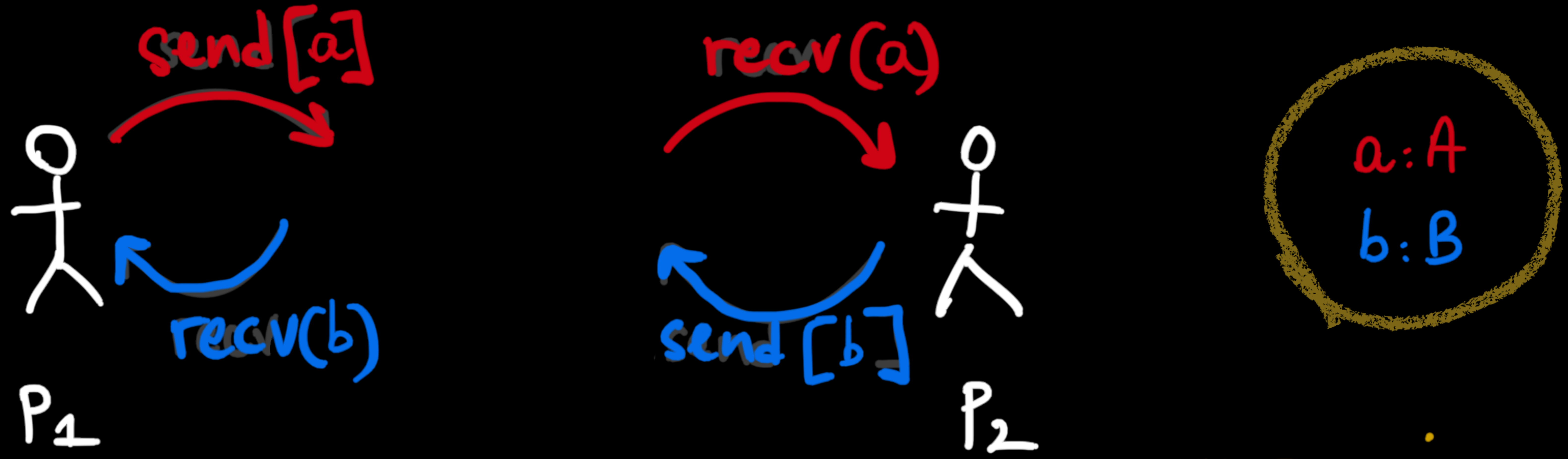
Binary Session Types



$P_1 = \text{send}[a].\text{recv}(b).\text{close}$

$P_2 = \text{recv}(a).\text{send}[b].\text{wait}$

Binary Session Types & Linear Logic



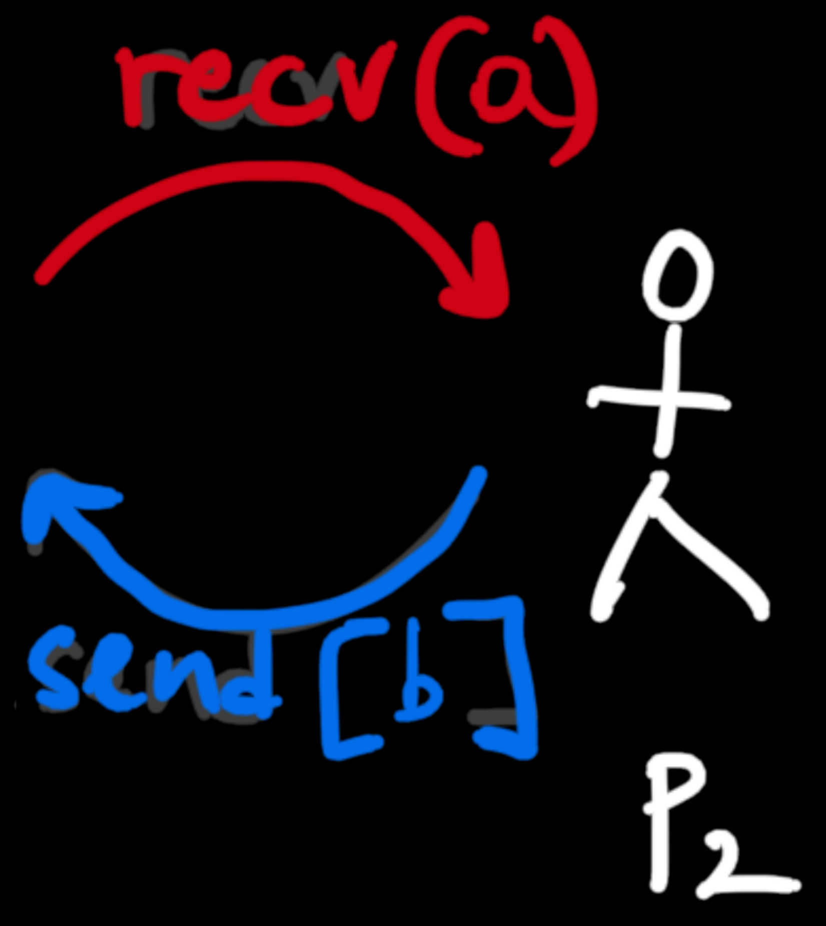
OUTPUT INPUT

$$P_1 = \text{send}[a].\text{recv}(b).\text{close} \quad \vdash \quad x_2 : A \otimes (B^{\perp} \parr 1)$$

$$P_2 = \text{recv}(a).\text{send}[b].\text{wait} \quad \vdash \quad x_1 : A^{\perp} \parr (B \otimes \perp)$$

[Caires & Pfenning / Wadler]

Proofs-as-processes



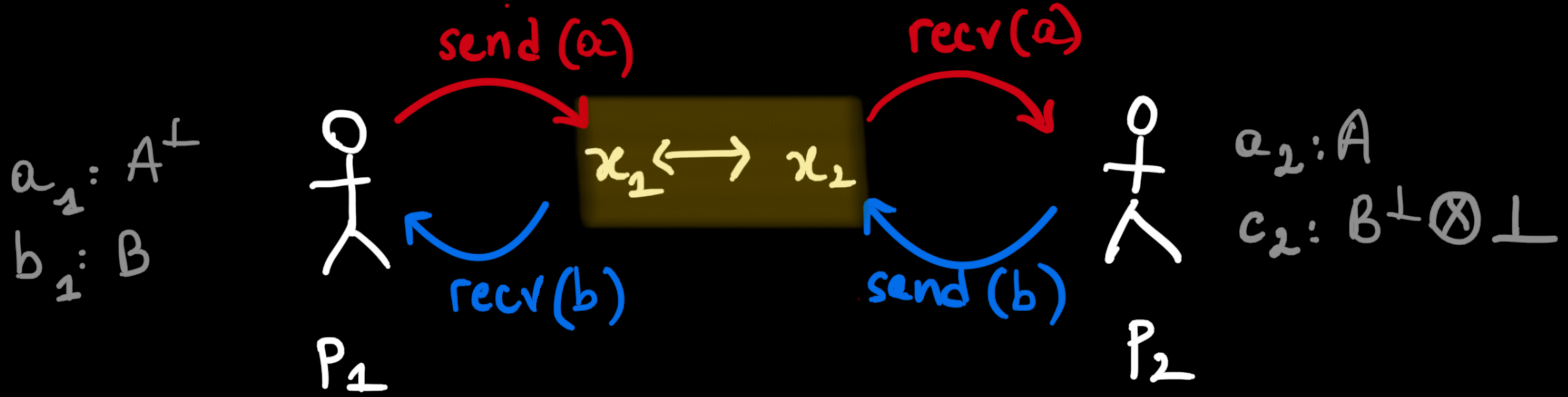
$$\dfrac{Q_1 = b \leftrightarrow b_1}{Q_1 \vdash b : B, b_1 : B^{\perp}}$$

$$\perp \; \dfrac{\dfrac{Q_2 = a \leftrightarrow a_1}{Q_2 \vdash a : A^{\perp}, a_1 : A}}{\text{wait}.Q_2 \vdash a : A^{\perp}, x_1 : \perp, a_1 : A}$$

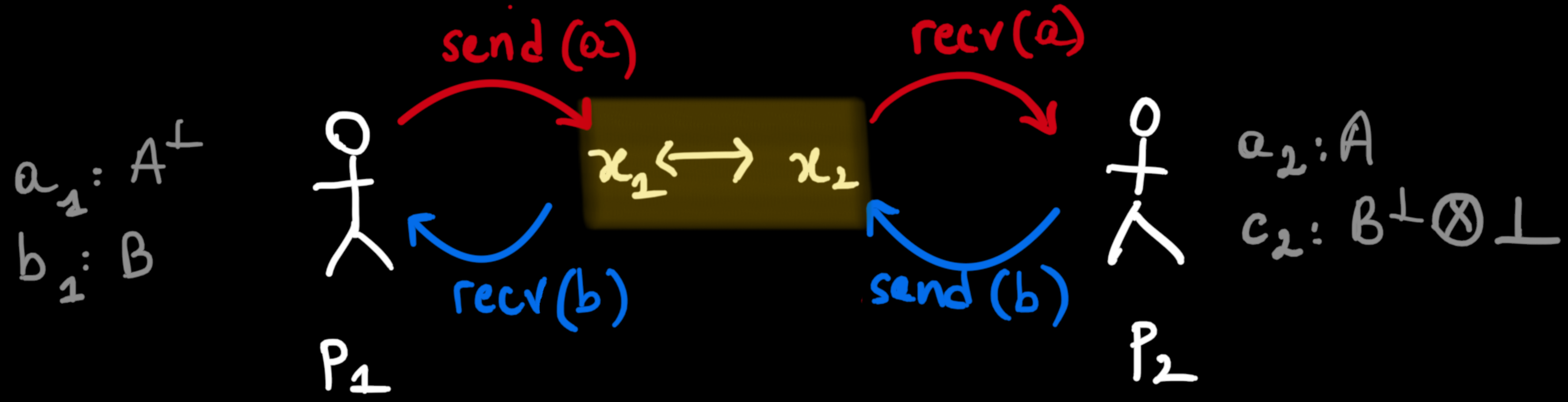
$$\otimes \; \dfrac{Q_1 \vdash b : B, b_1 : B^{\perp} \qquad \text{wait}.Q_2 \vdash a : A^{\perp}, x_1 : \perp, a_1 : A}{\text{send}[b \triangleright Q_1].\text{wait}.Q_2 \vdash a : A^{\perp}, x_1 : B \otimes \perp, a_1 : A, b_1 : B^{\perp}}$$

$$⅋ \; \dfrac{\text{send}[b \triangleright Q_1].\text{wait}.Q_2 \vdash a : A^{\perp}, x_1 : B \otimes \perp, a_1 : A, b_1 : B^{\perp}}{P_1 = \text{recv}(a).\text{send}[b \triangleright Q_1].\text{wait}.Q_2 \vdash x_1 : A^{\perp} ⅋ (B \otimes \perp), a_1 : A, b_1 : B^{\perp}}$$

Reduction-as-communication



Reduction-as-communication



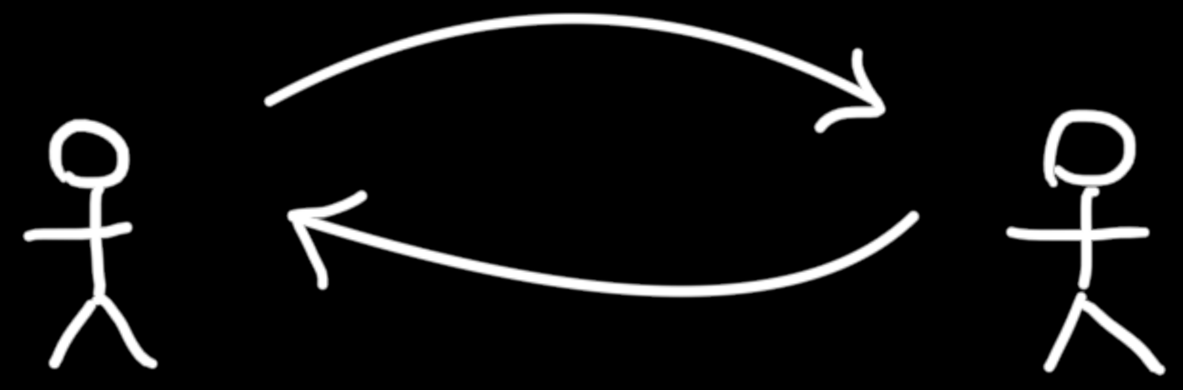
$$\dfrac{P_1 \vdash x_1 : A \otimes B^\perp \parr 1 \qquad P_2 \vdash x_2 : A^\perp \parr B \otimes \perp}{(\nu x_1 x_2)\,(P_1 \,\|\, P_2) \vdash \cdot}\ \text{CUT}$$

dual types!

Generalizing CUT

Synchronous

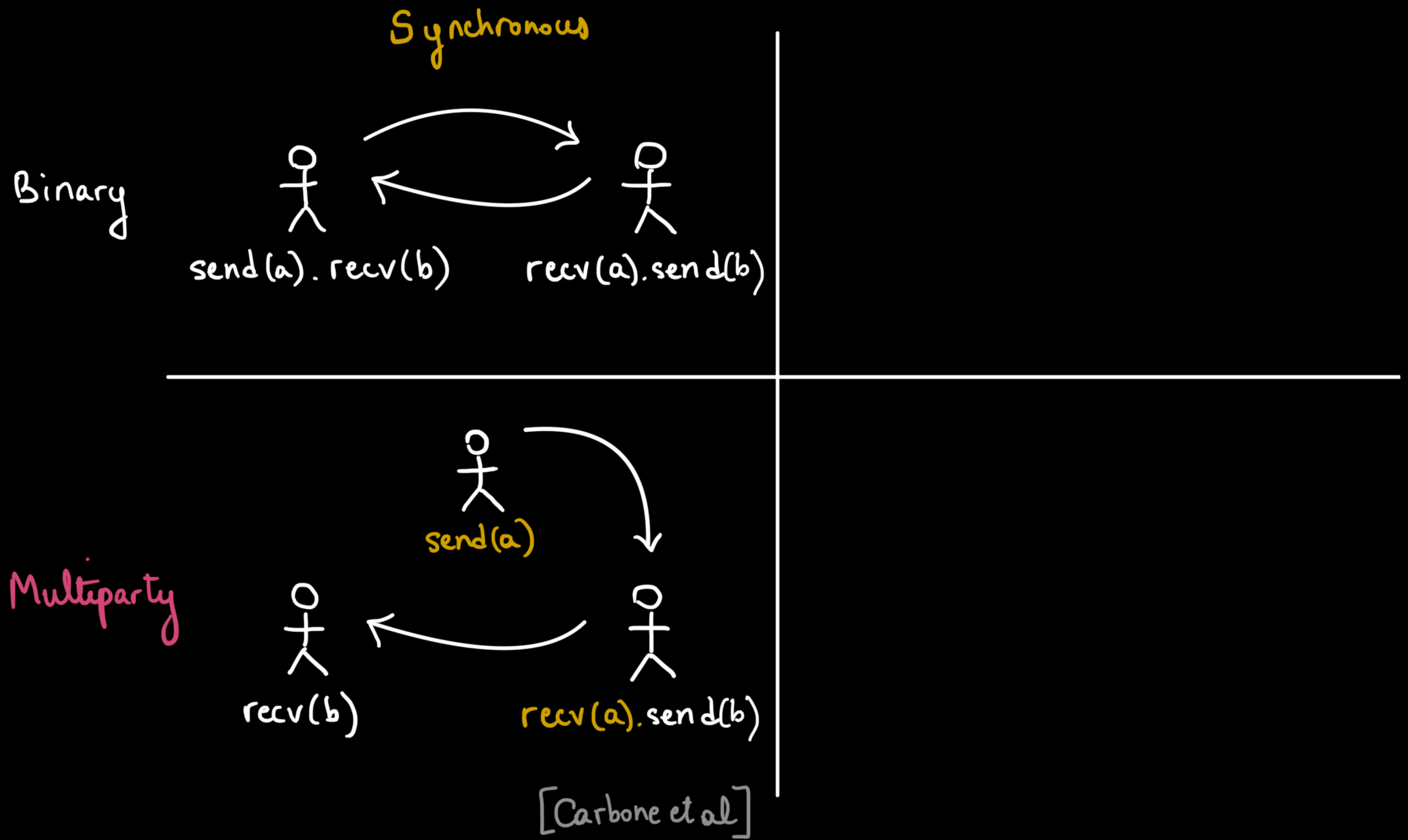
Binary



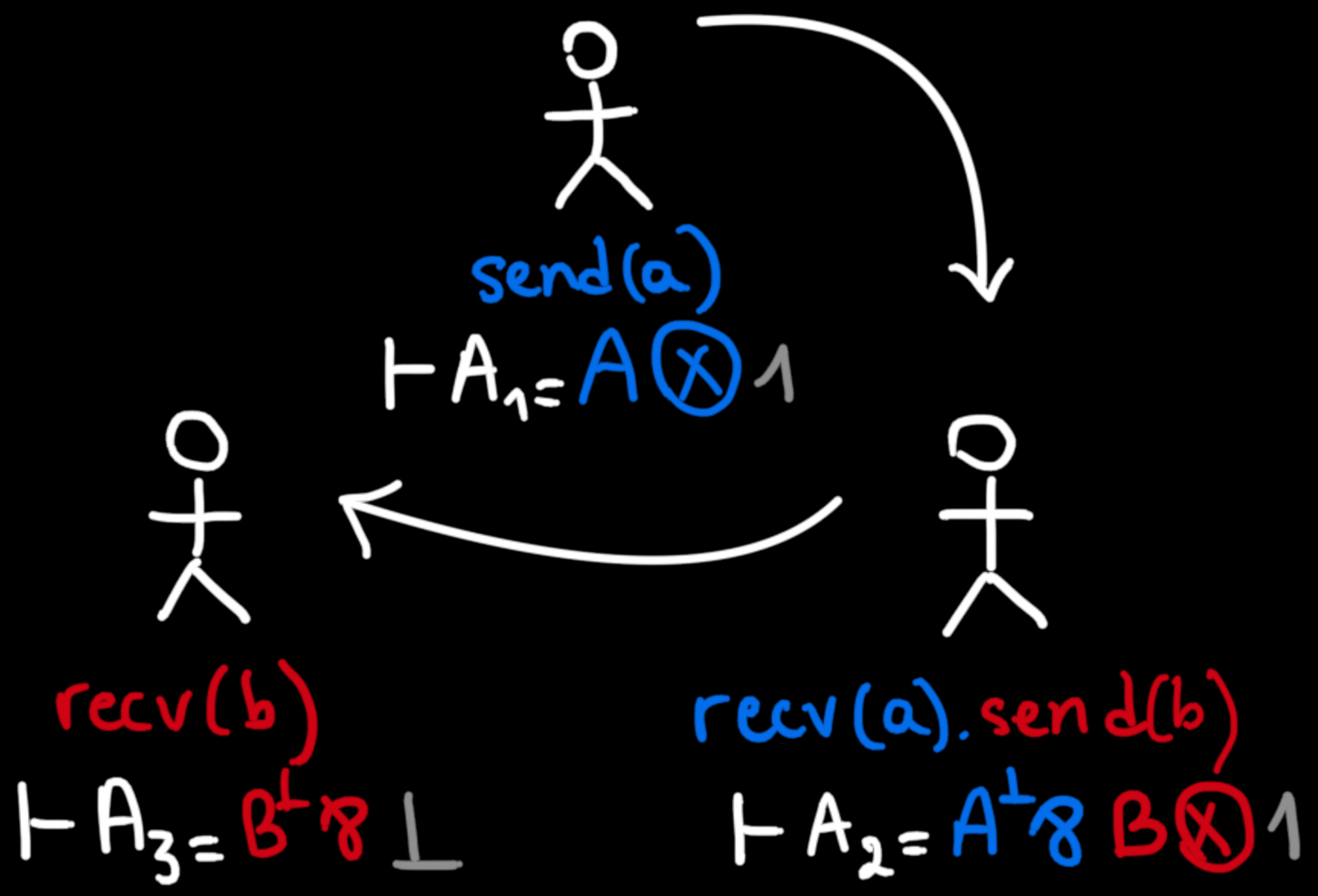
send(a). recv(b)

recv(a). send(b)

Generalizing CUT

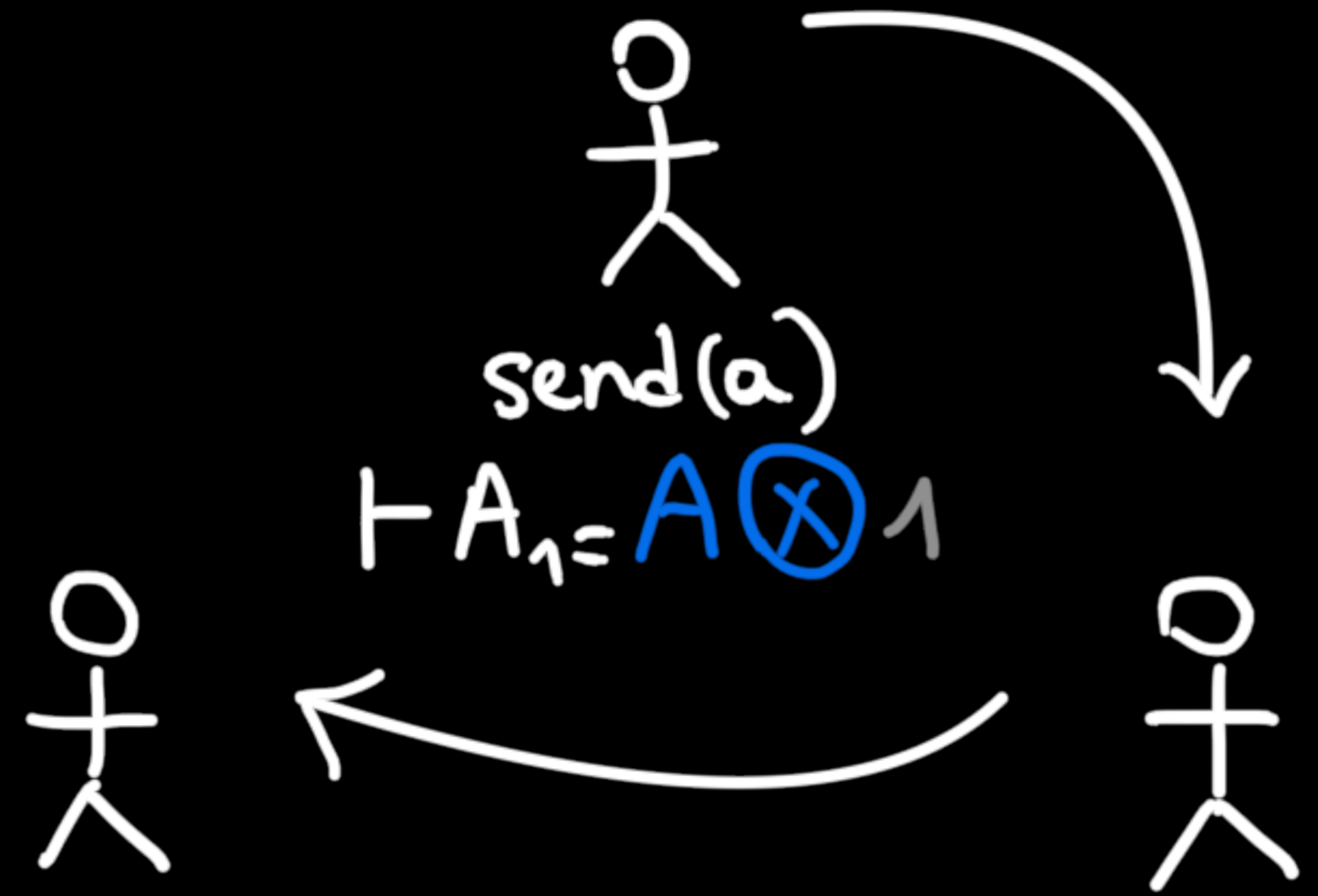


Coherence



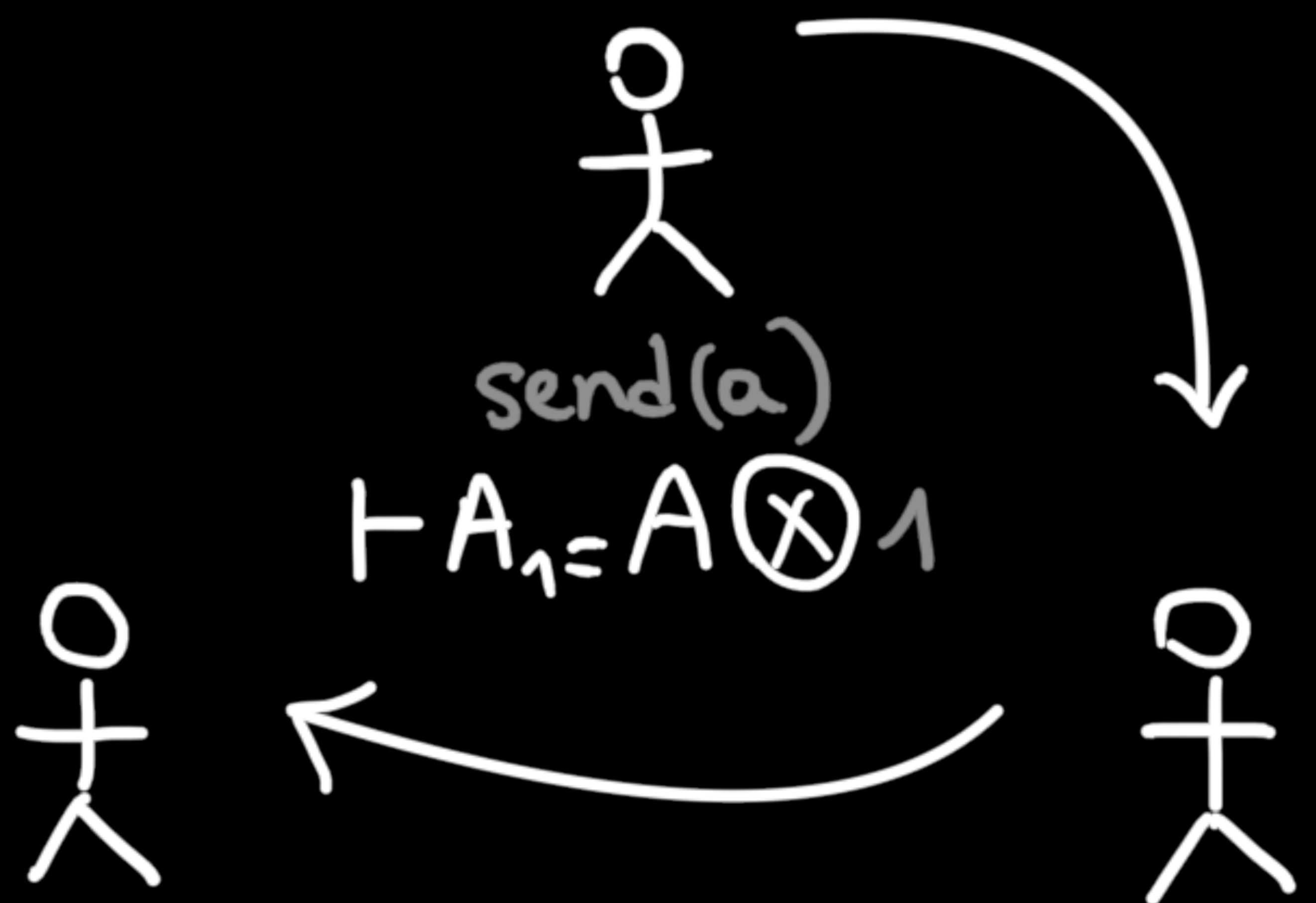
send(a)
$\vdash A_1 = A \otimes 1$
recv(b)
$\vdash A_3 = B^{\perp} ⅋ \perp$
recv(a).send(b)
$\vdash A_2 = A^{\perp} ⅋ B \otimes 1$

Coherence



$$\dfrac{\begin{array}{c}\mathrm{send}(a)\\ \vdash A_1 = A \otimes 1\end{array} \qquad \begin{array}{c}\mathrm{recv}(b)\\ \vdash A_3 = B^\perp \parr \bot\end{array} \qquad \begin{array}{c}\mathrm{recv}(a).\mathrm{send}(b)\\ \vdash A_2 = A^\perp \parr B \otimes 1\end{array} \qquad \mathrm{coh}(A_1, A_2, A_3)}{(\nu x_1 x_2 x_3)(P_1 \mid P_2 \mid P_3) \vdash \cdot}\ \mathrm{GCUT}$$

Coherence



send(a)

$\vdash A_1 = A \otimes 1$

recv(b)

$\vdash A_3 = B^\perp \parr \bot$

recv(a).send(b)

$\vdash A_2 = A^\perp \parr B \otimes 1$

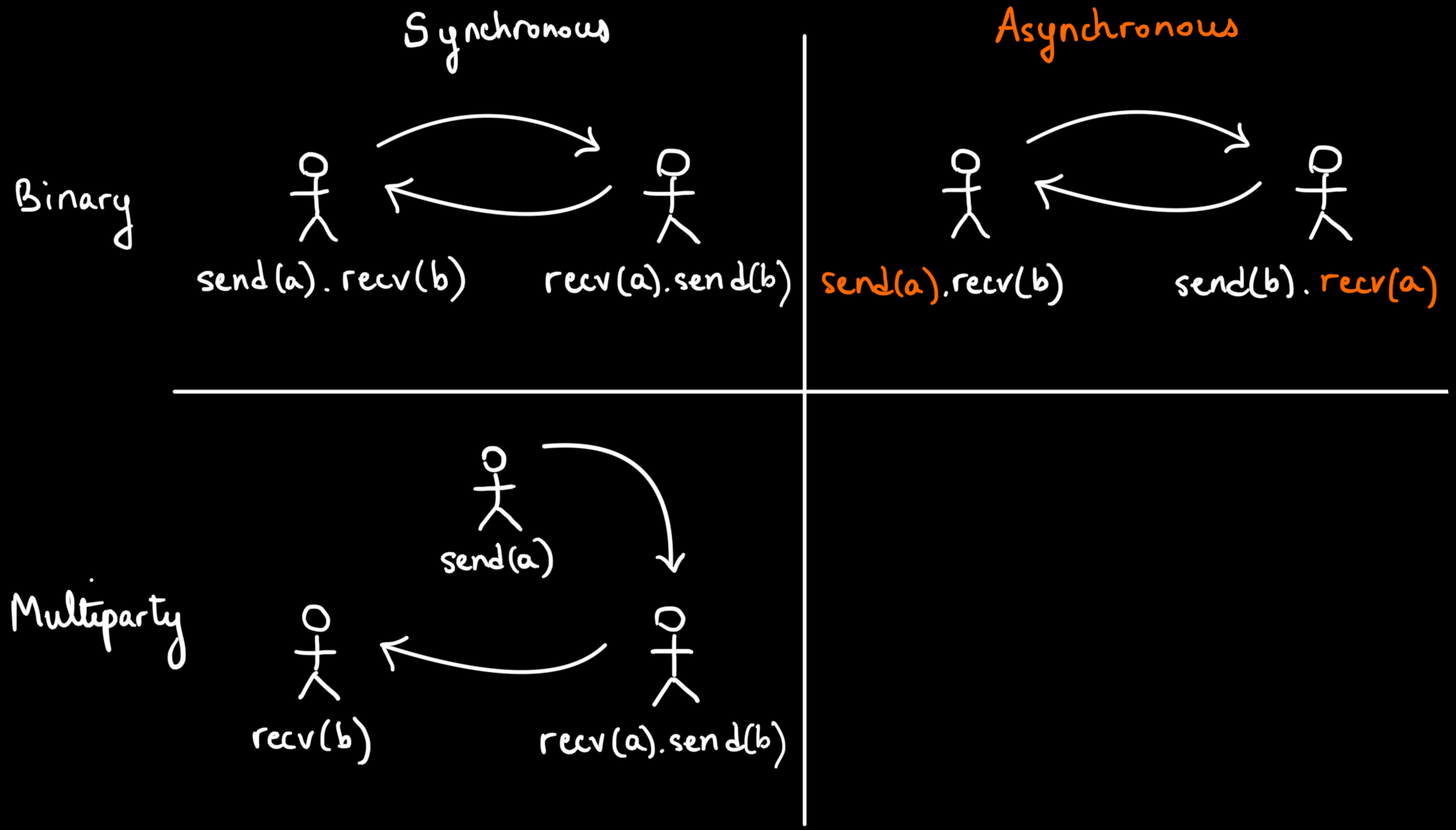
$$\dfrac{\dfrac{1 \,,\quad 1 \quad,\quad \bot}{1 \,,\quad B \otimes 1 \,,\quad B^\perp \parr \bot}}{A \otimes 1 \,,\quad A^\perp \parr B \otimes 1 \,,\quad B^\perp \parr \bot}$$

COHERENCE

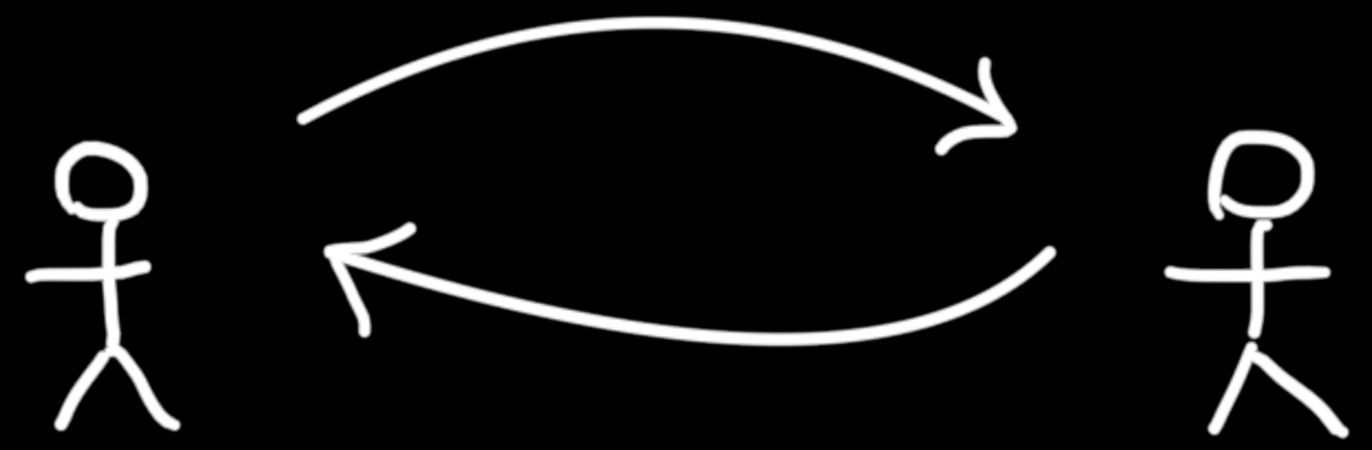
$$\dfrac{\vdash A_3 = B^\perp \parr \bot \qquad \vdash A_2 = A^\perp \parr B \otimes 1 \qquad \dfrac{\Pi}{coh(A_1, A_2, A_3)}}{(\nu x_1 x_2 x_3)(P_1 \mid P_2 \mid P_3) \vdash \cdot}\ \text{GCUT}$$

[Carbone et al]

Generalizing CUT



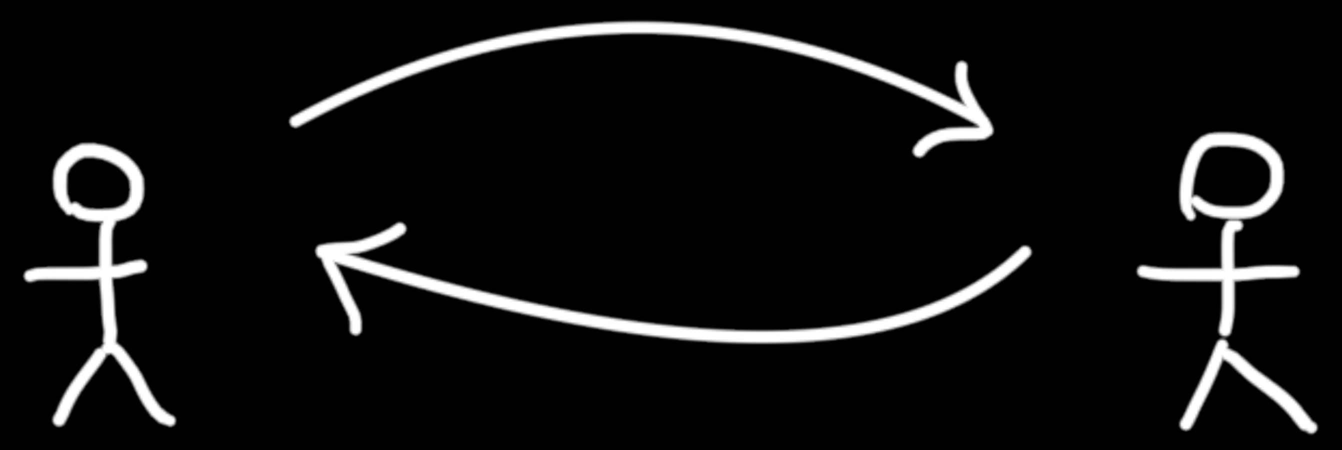
Criss-cross



send(a).recv(b) $\qquad\qquad$ send(b).recv(a)

$\vdash A \otimes B^{\perp} \parr 1 \qquad\qquad \vdash B \otimes A^{\perp} \parr \bot$

Criss-cross

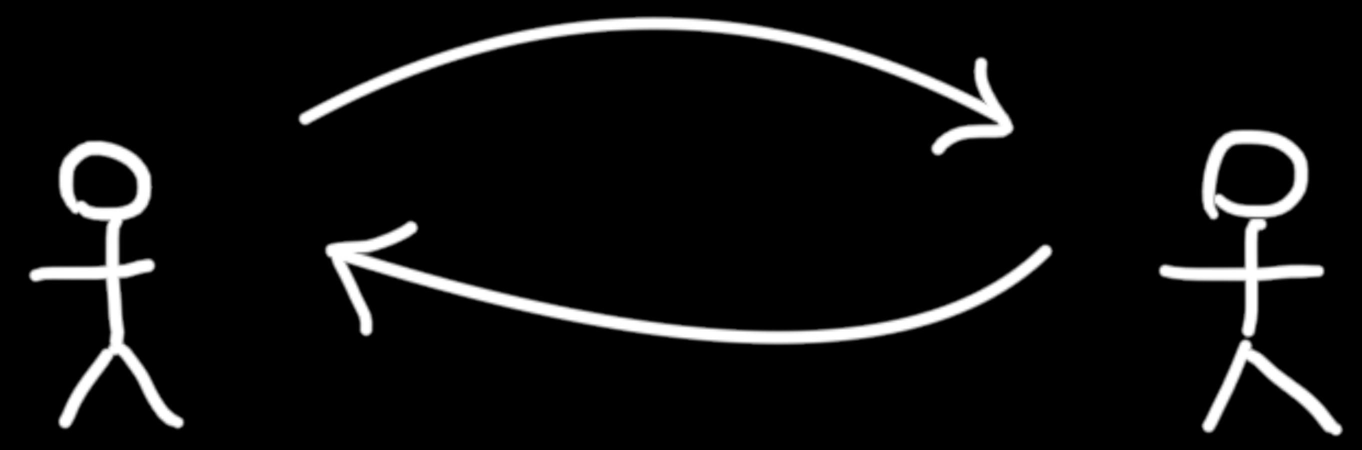


$$\frac{\begin{matrix} \text{send}(a).\text{recv}(b) & & \text{send}(b).\text{recv}(a) & & \Pi \\ \vdash A_1 = A \otimes B^{\perp} ⅋ 1 & & \vdash A_2 = B \otimes A^{\perp} ⅋ \perp & & \text{comp}(A_1, A_2) \end{matrix}}{(\nu x_1 x_2)(P_1 \mid P_2) \vdash \cdot}$$

Criss-cross

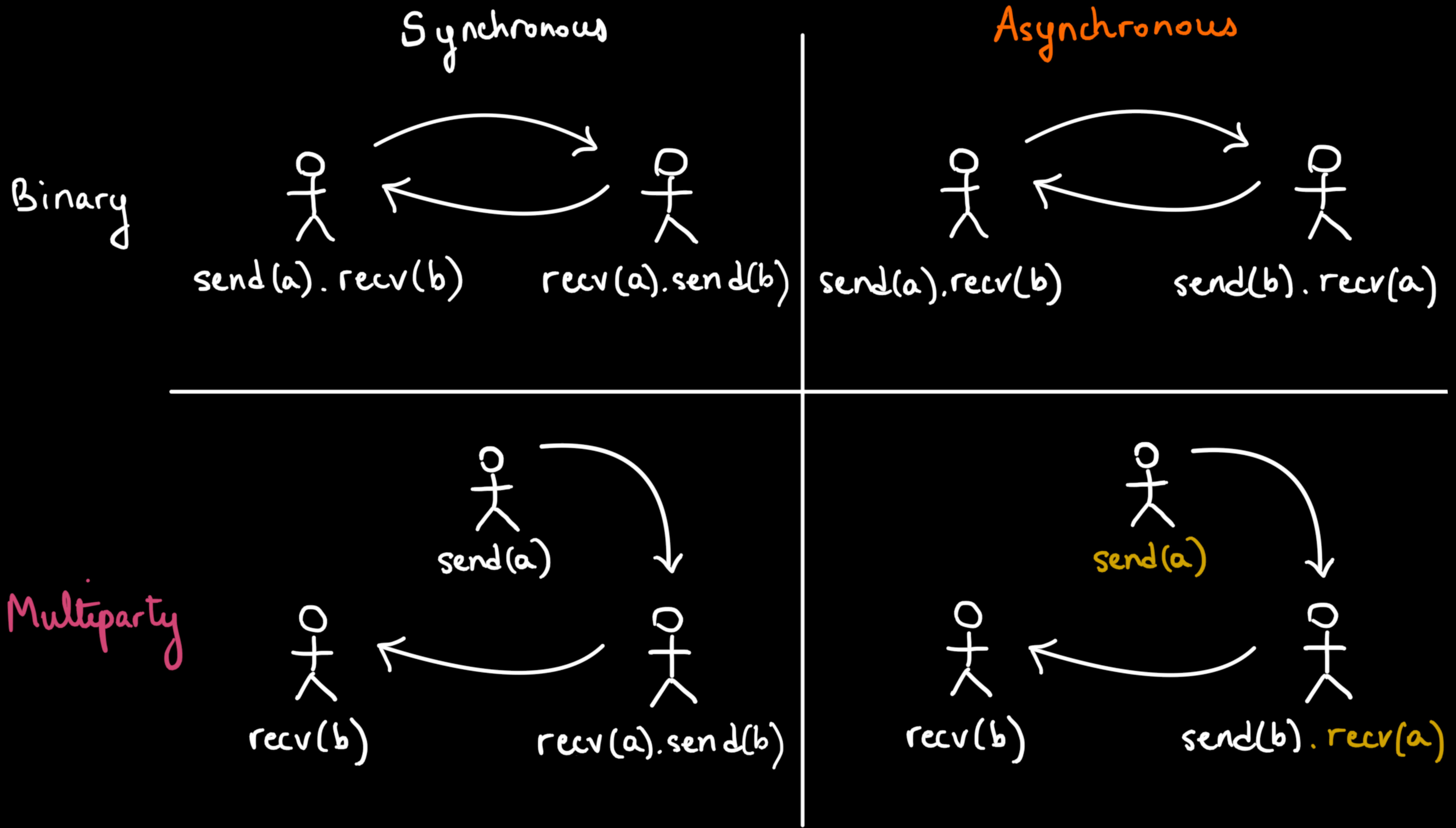
$$\dfrac{\dfrac{\perp,\ 1}{[A^\perp]\ B\otimes\perp,\ [B^\perp]\ A\otimes 1}}{A^\perp \parr B\otimes\perp,\ B^\perp\parr A\otimes 1}$$

via BUFFERING

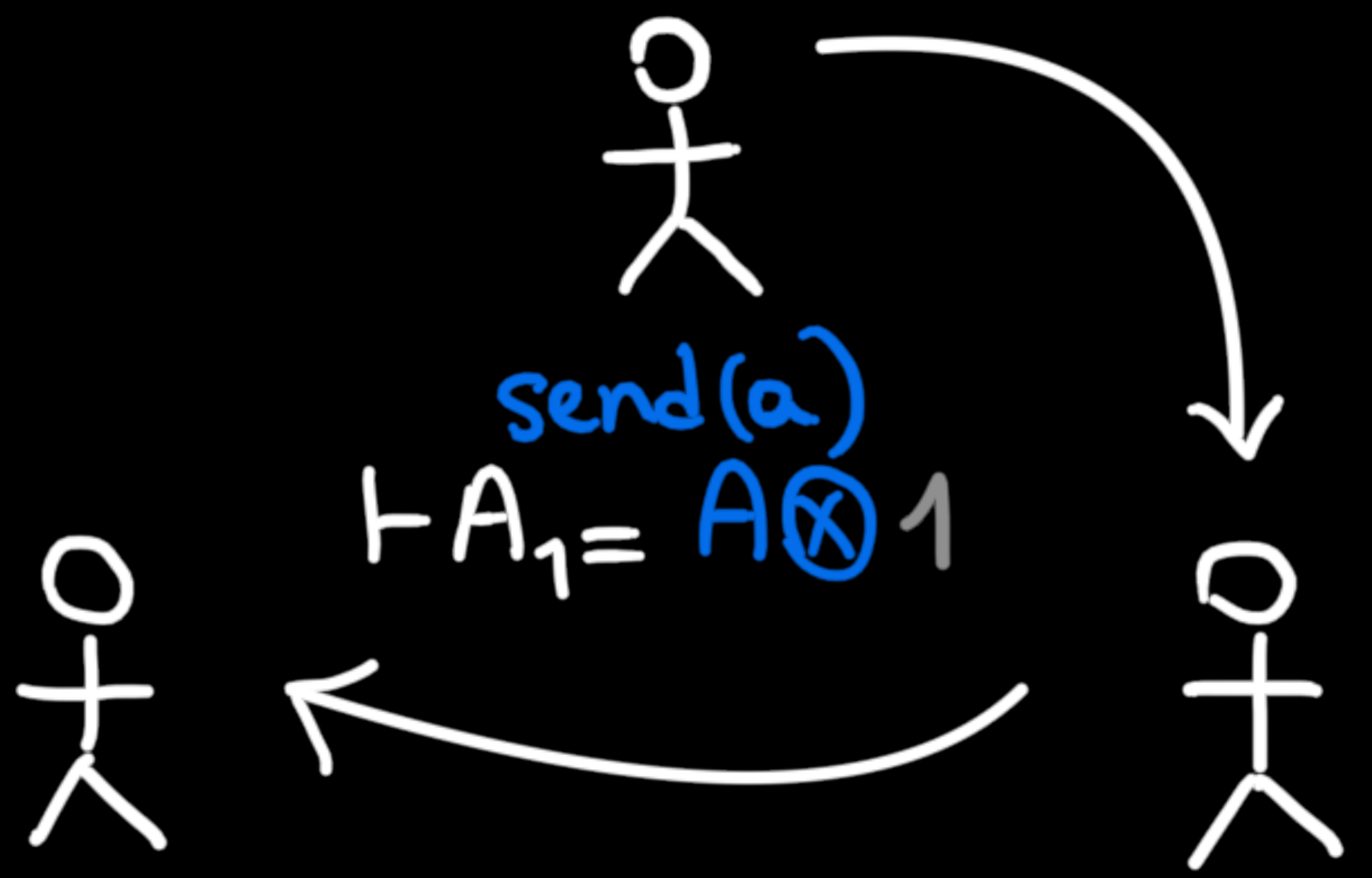


$$\dfrac{\begin{array}{ccc} \text{send}(a).\text{recv}(b) & \text{send}(b).\text{recv}(a) & \\ \vdash A_1 = A\otimes B^\perp \parr 1 & \vdash A_2 = B\otimes A^\perp \parr \perp & \text{comp}(A_1^\perp, A_2^\perp) \end{array}}{(\nu x_1 x_2)(P_1 \mid P_2) \vdash \cdot}$$

Generalizing CUT



Forwarders



send(a)

$\vdash A_1 = A \otimes 1$

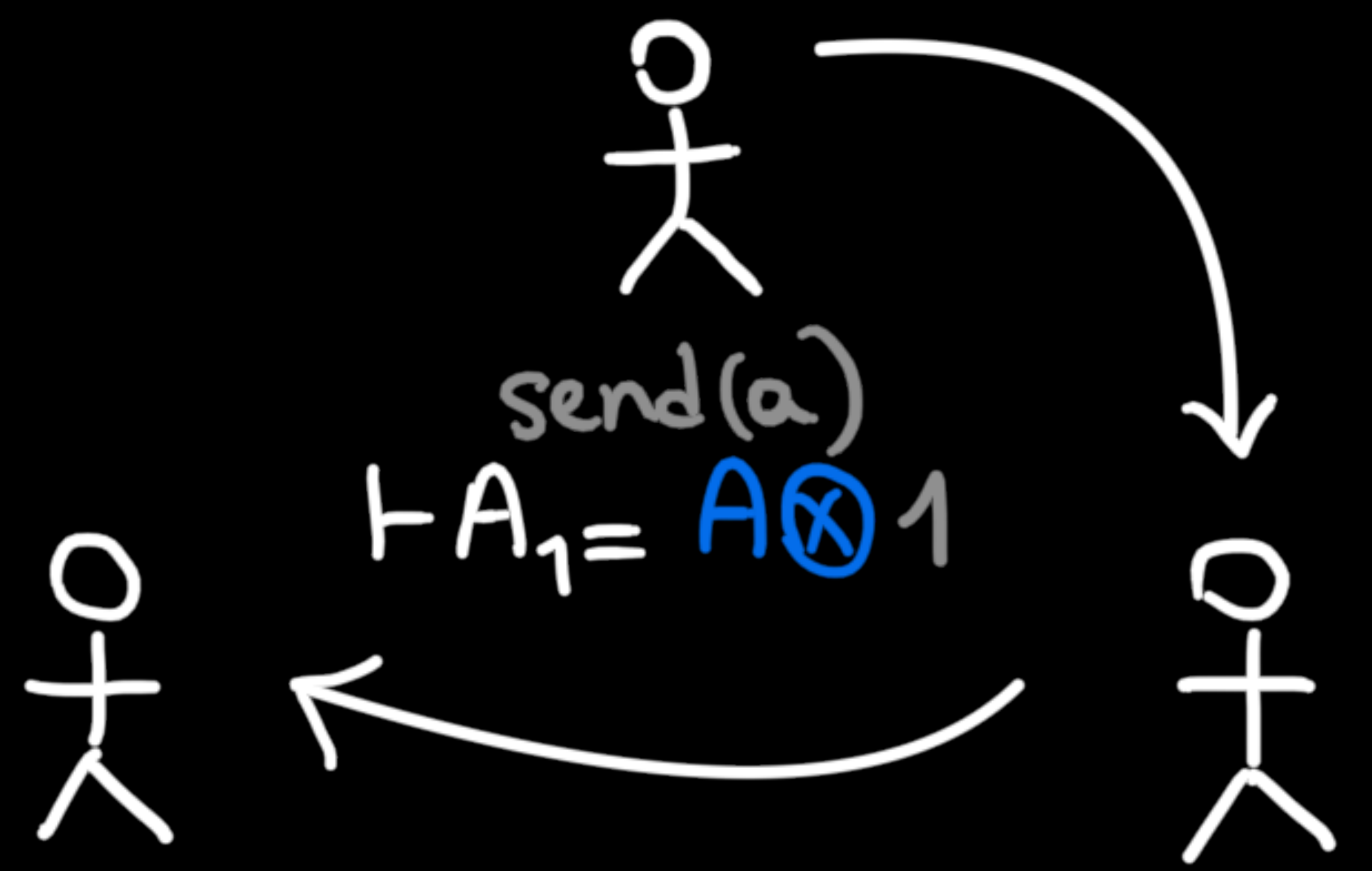
recv(b)

$\vdash A_3 = B^\perp \parr \perp$

send(b) . recv(a)

$\vdash A_2 = B \otimes A^\perp \parr 1$

Forwarders



send(a)

$\vdash A_1 = A \otimes 1$

recv(b)

$\vdash A_3 = B^{\perp} ⅋ \perp$

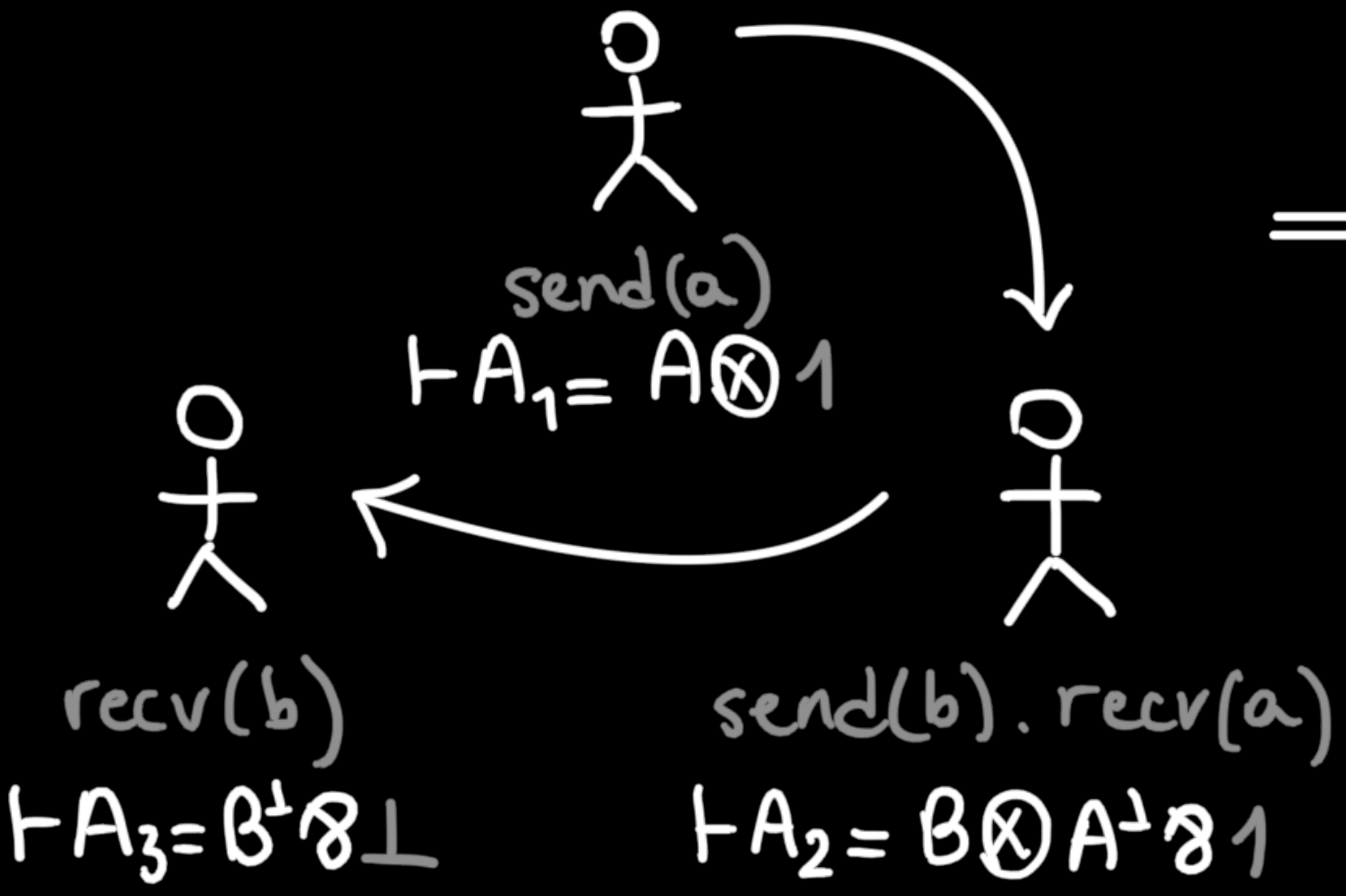
send(b) . recv(a)

$\vdash A_2 = B \otimes A^{\perp} ⅋ 1$

fwd (A_1, A_2, A_3)

$$\frac{\vdash A_1 \qquad \vdash A_2 \qquad \vdash A_3 \qquad \text{fwd}(A_1, A_2, A_3)}{(\nu x_1 x_2 x_3)(P_1 \mid P_2 \mid P_3) \vdash \cdot}$$

Forwarders



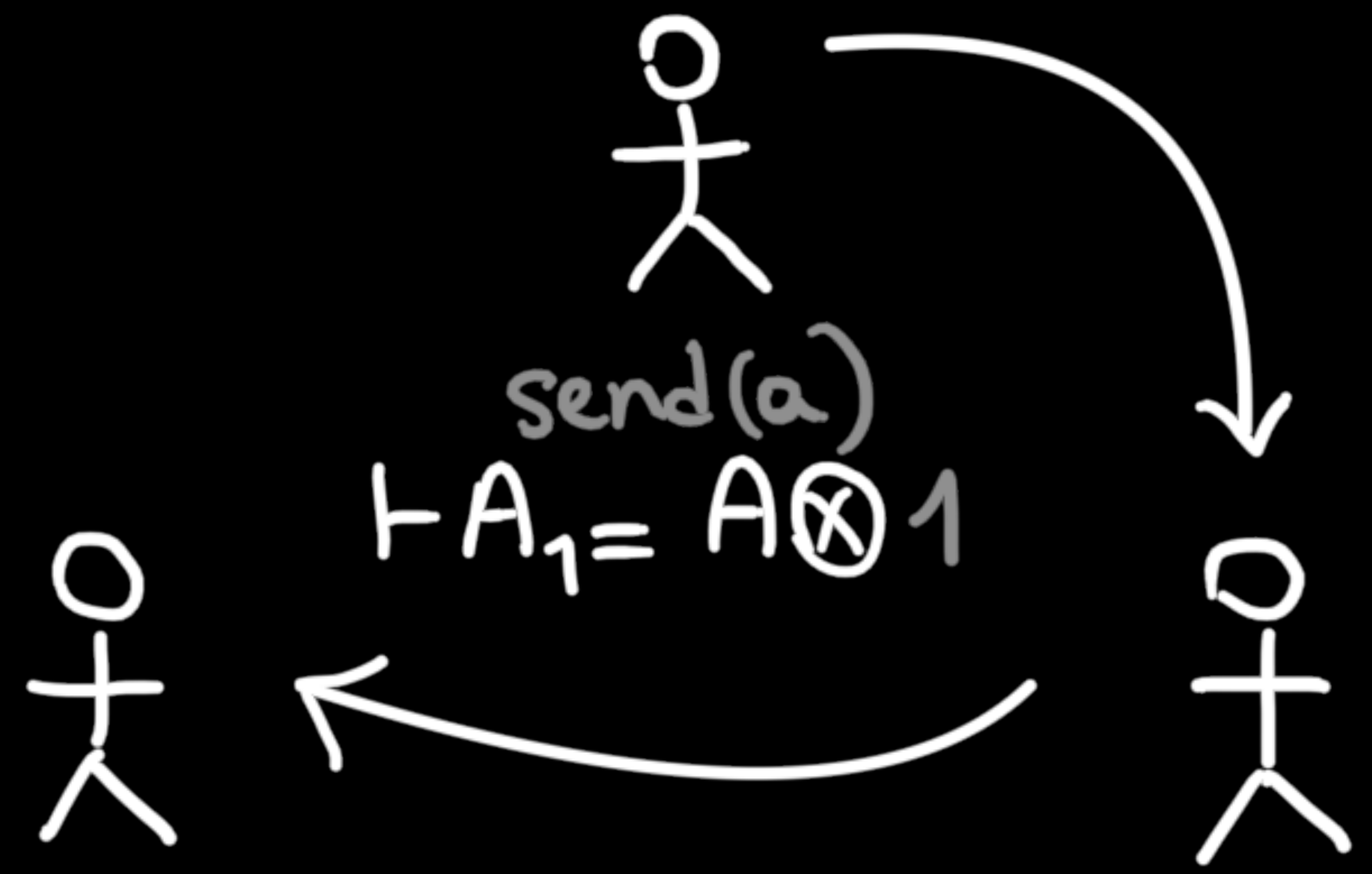
$$\dfrac{\left[A^\perp\right]^{x_2}\ x_1 : \perp,\ \left[B^\perp\right]^{x_3}\ x_2 : A \otimes \perp,\ x_3 : B \otimes 1}{x_1 : A^\perp \parr \perp,\ x_2 : B^\perp \parr A \otimes \perp,\ x_3 : B \otimes 1}$$

x_2 $\quad$ x_3

fwd $(A_1^\perp, A_2^\perp, A_3^\perp)$

$$(\nu x_1 x_2 x_3)(P_1 \mid P_2 \mid P_3) \vdash \cdot$$

Forwarders



send(a)

$\vdash A_1 = A \otimes 1$

recv(b)

$\vdash A_3 = B^\perp ⅋ \perp$

send(b) . recv(a)

$\vdash A_2 = B \otimes A^\perp ⅋ 1$

$$\dfrac{\dfrac{x_1 : \perp, [B^\perp]^{x_3}\, x_2 : \perp,\ x_3 : B \otimes 1}{[A^\perp]^{x_2}\, x_1 : \perp, [B^\perp]\, x_2 : A \otimes \perp,\ x_3 : B \otimes 1}}{x_1 : A^\perp ⅋ \perp,\ x_2 : B^\perp ⅋ A \otimes \perp,\ x_3 : B \otimes 1}$$

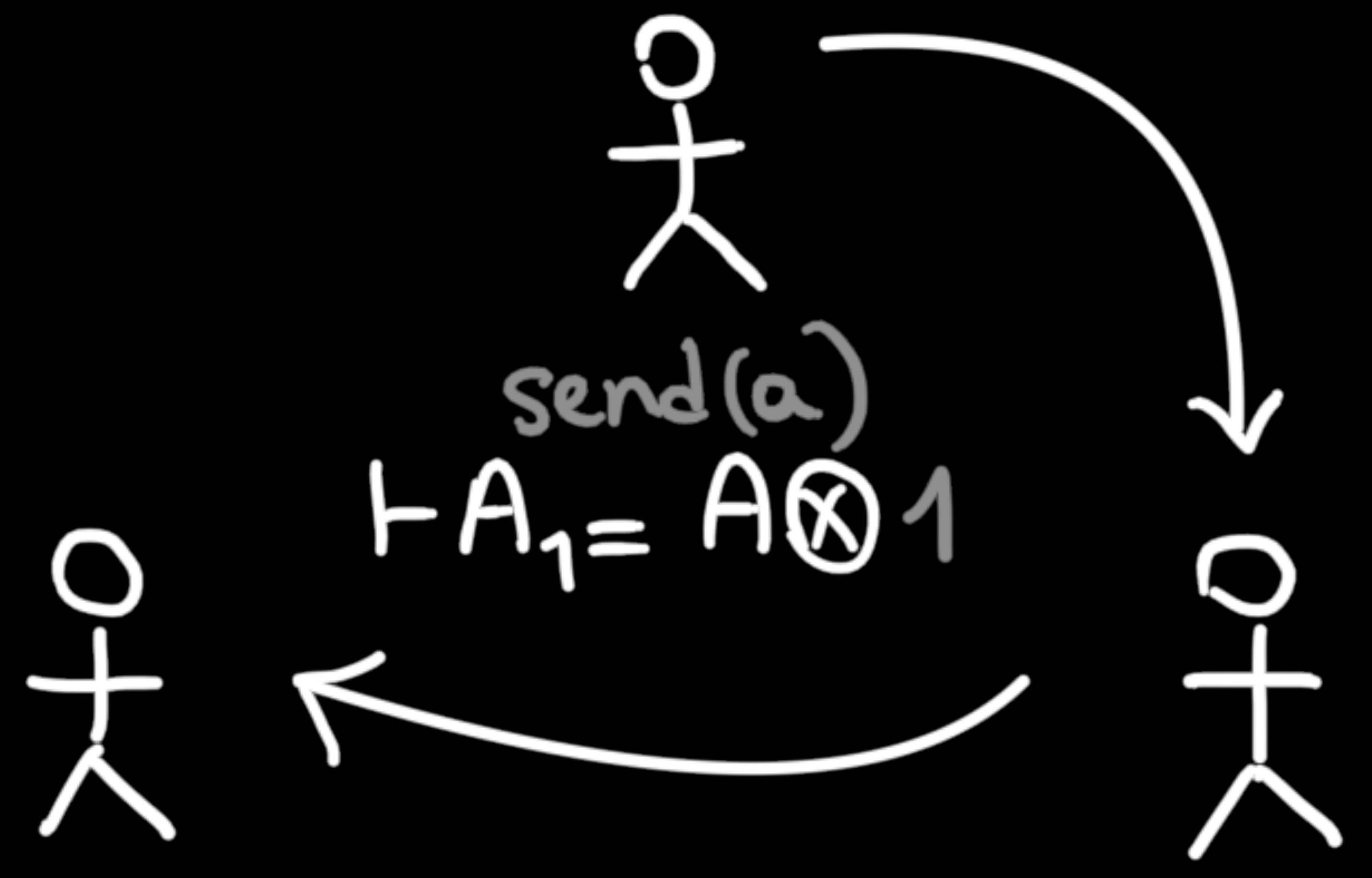
$x_1 \qquad x_2$

$\Vert$

$\mathrm{fwd}\,(A_1^\perp, A_2^\perp, A_3^\perp)$

$$\dfrac{}{(\nu x_1 x_2 x_3)(P_1 \mid P_2 \mid P_3) \vdash \cdot}$$

Forwarders



send(a) $\vdash A_1 = A \otimes 1$

recv(b) $\vdash A_3 = B^\perp \parr \bot$

send(b) . recv(a) $\vdash A_2 = B \otimes A^\perp \parr 1$

$$\dfrac{\vdash A_3 = B^\perp \parr \bot \qquad \vdash A_2 = B \otimes A^\perp \parr 1 \qquad \vdash A_1 = A \otimes 1 \qquad \mathrm{fwd}(A_1^\perp, A_2^\perp, A_3^\perp)}{(\nu x_1 x_2 x_3)(P_1 | P_2 | P_3) \vdash \cdot}$$

$$\mathrm{fwd}(A_1^\perp, A_2^\perp, A_3^\perp) = $$

$$\dfrac{\dfrac{\dfrac{x_1 : \bot^{x_3}, \quad x_2 : \bot^{x_3}, \quad x_3 : 1^{x_1 x_2}}{x_1 : \bot, \ [B^\perp]^{x_3} \ x_2 : \bot, \quad x_3 : B \otimes^{x_2} 1}}{[A^\perp]^{x_2} \ x_1 : \bot, \ [B^\perp]^{x_3} \ x_2 : A \otimes^{x_1} \bot, \ x_3 : B \otimes 1}}{x_1 : A^\perp \parr^{x_2} \bot, \ x_2 : B^\perp \parr^{x_3} A \otimes \bot, \ x_3 : B \otimes 1}$$

ASYNCHRONOUS FORWARDERS

Multiparty compatibility – logically

Asynchronous forwarders characterize precisely

the multiparty compatible types.

semantic notion from [Denielou & Yoshida]

refined by several authors

e.g. [Scalas et al.]

Multiparty compatibility – logically

Asynchronous forwarders characterize precisely

the multiparty compatible types.

and still satisfy cut-elimination

i.e. $(\nu x_1 x_2)(F_1 \parallel F_2)$ is an async. forwarder

arxiv.org/abs/2112.07636

Thank you!