Certified Abstract Machines for Skeletal Semantics

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Defining a Language on Paper

Example: Call-by-Value λ-calculus

Variables	$x \in \mathcal{V}$
Term	$t ::= \; x \mid t \; t \mid \lambda x.t$
Closure	$c ::= \; (x, t, s)$
Environment	$s ::= \; [(x_1 \mapsto c_1), \dots, (x_n \mapsto c_n)]$

$$\frac{s(x) = c}{s, x \Downarrow c} \qquad\qquad \frac{}{s, \lambda x.t \Downarrow (x, t, s)}$$

$$\frac{s, t_1 \Downarrow (x, t, s') \qquad s, t_2 \Downarrow c' \qquad (s' + \{x \mapsto c'\}), t \Downarrow c}{s, (t_1 \;\, t_2) \Downarrow c}$$

Defining a Language with a Computer

In a proof assistant, from scratch

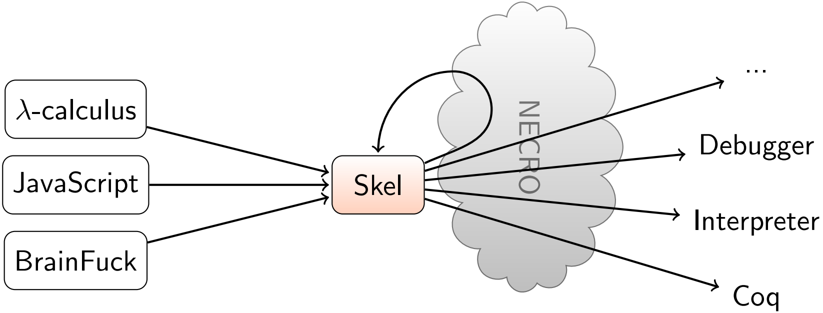
- Coq
- Isabelle/HOL
- Agda, Twelf, . . .

In a convenient Framework

- Ott, Lem
- $\mathbb{K}$
- Skeletal Semantics

Skeletal Semantics

- Recent framework (first definition: POPL 2019)
- Meta-language (Skel) to define programming languages
- Toolbox to manipulate semantics: Necro.



Skeletal Semantics for CbV λ-calculus

```
type ident

type lterm =
| Lam (ident, lterm)
| Var ident
| App (lterm, lterm)

type clos =
| Clos (ident, lterm, env)

type env

term extEnv: (env,ident,clos) → env
term getEnv: (ident,env) → clos
```

```
term eval (s:env) (l:lterm): clos =
branch
  let Lam (x, t) = l in
  Clos (x, t, s)
or
  let Var x = l in
  getEnv (x, s)
or
  let App (t1, t2) = l in
  let Clos (x, t, s') = eval s t1 in
  let w = eval s t2 in
  let s'' = extEnv (s', x, w) in
  eval s'' t
end
```

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Unspecified Types

We do not explicit what the elements look like.

E.g., there exist variables.

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Specified Types

Defined as algebraic data-types with constructors.

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Unspecified Terms

For when the actual implementation is not important.

E.g., we can extend an environment, and we can read the mapping of a variable.

Skeletal Semantics for CbV λ-calculus

Specified Term

Evaluation functions we want to describe.

There are associated with a given definition.

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```

Skeletal Semantics for CbV λ-calculus

Branching

Construction of the meta-language to list several possible behaviors.

Can be used to represent pattern-machings (like here), conditional statements, non-deterministic choices, etc.

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end
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Semantics of Skel?

Main semantics of Skel is Big-Step.

Wish for a different format of semantics: Abstract Machines.
Notably, would like an executable semantics.

For this, known technique by Danvy et al.:

- CPS Transform
- Defunctionalization

Non-Deterministic Abstract Machine

$$\langle \texttt{let}\ \ p = S_1\ \ \texttt{in}\ \ S_2, \kappa\rangle_{\mathsf{sk}} \to \langle S_1, \lceil \texttt{let}\ \ p = \Box\ \ \texttt{in}\ \ S_2 \rfloor :: \kappa\rangle_{\mathsf{sk}}$$

$$\langle \texttt{Branch}(l), \kappa\rangle_{\mathsf{sk}} \to \langle S, \kappa\rangle_{\mathsf{sk}} \quad \texttt{for}\ \ (S \in l)$$

$$\cdots \to \cdots$$

Non-Deterministic Abstract Machine

$$\langle \texttt{let}\ \ p = S_1\ \ \texttt{in}\ \ S_2, \kappa\rangle_{\mathsf{sk}} \to \langle S_1, \lceil \texttt{let}\ \ p = \Box\ \ \texttt{in}\ \ S_2 \rfloor :: \kappa\rangle_{\mathsf{sk}}$$
$$\langle \texttt{Branch}(l), \kappa\rangle_{\mathsf{sk}} \to \langle S, \kappa\rangle_{\mathsf{sk}} \quad \texttt{for}\ \ (S \in l)$$
$$\cdots \to \cdots$$

Problem: still non-deterministic, so not really computable...
Next: deterministic AM, with backtracking.

Deterministic Abstract Machine

$$\langle \texttt{let}\ \ p = S_1\ \ \texttt{in}\ \ S_2, \kappa, f\rangle_{\mathsf{sk}} \to \langle S_1, \lceil \texttt{let}\ \ p = \Box\ \ \texttt{in}\ \ S_2 \rfloor :: \kappa, f\rangle_{\mathsf{sk}}$$

$$\langle \texttt{Branch}(S :: l), \kappa, f\rangle_{\mathsf{sk}} \to \langle S, \kappa, \llbracket \texttt{Branch}(l), \kappa \rrbracket :: f\rangle_{\mathsf{sk}}$$

$$\langle \texttt{Branch}([]), \kappa, f\rangle_{\mathsf{sk}} \to \langle f\rangle_{\mathsf{fk}}$$

$$\cdots \to \cdots$$

$$\langle \llbracket S, \kappa \rrbracket :: f\rangle_{\mathsf{fk}} \to \langle S, \kappa, f\rangle_{\mathsf{sk}}$$

Equivalence Certification

Definitions in Coq:

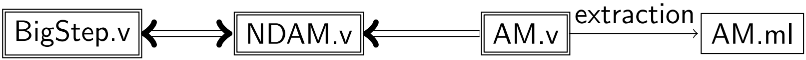
- Big-Step semantics already defined
- We define the Non-Deterministic Abstract Machine
  ```
  Inductive step: state -> state -> Prop
  ```
- We define the Deterministic Abstract Machine
  ```
  Definition step (a: state) : option state
  ```

Certification:

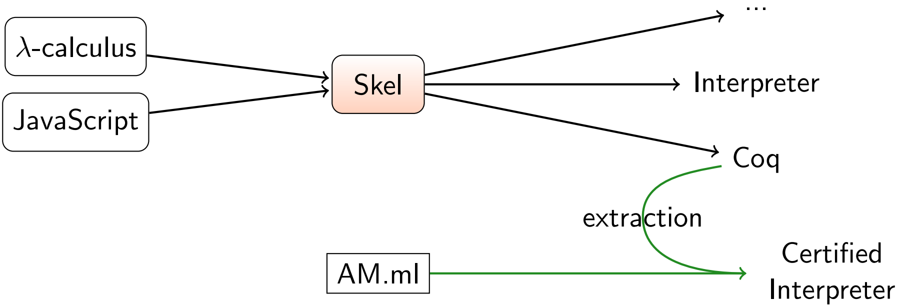
- We prove Big-Step and NDAM are equivalent (standard proof)
- We prove AM is sound w.r.t. NDAM (cut backtracks)

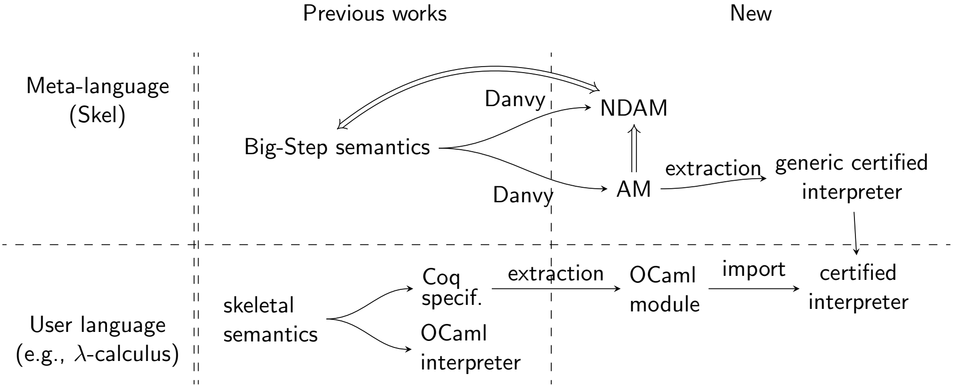
Certified Interpreter

Now we have different semantics for Skel:



For the user, we can produce a certified interpreter:





Previous works

New

Meta-language (Skel)

User language (e.g., λ-calculus)

Big-Step semantics

Danvy

NDAM

Danvy

AM

extraction

generic certified interpreter

skeletal semantics

Coq specif.

OCaml interpreter

extraction

OCaml module

import

certified interpreter

Syntax of Skel

$$\begin{array}{rl} \text{Identifier} & x \in \mathcal{V} \\ \text{Term} & t ::= x \mid C\ t \mid (t, \ldots, t) \mid \lambda p.S \\ \text{Skeleton} & S ::= t_0\ t_1 \ldots t_n \mid \texttt{let}\ p = S_1\ \texttt{in}\ S_2 \\ & \qquad \mid \texttt{Branch}(S, \ldots, S) \mid t \\ \text{Pattern} & p ::= \_ \mid x \mid C\ p \mid (p, \ldots, p) \end{array}$$

Non-Deterministic Abstract Machine

$$\begin{aligned}
\langle \texttt{Branch}(l), \Sigma, \kappa\rangle_{\mathsf{sk}} &\to \langle S, \Sigma, \kappa\rangle_{\mathsf{sk}} \quad \texttt{for}\ (S \in l) \\
\langle \texttt{let}\ p = S_1\ \texttt{in}\ S_2, \Sigma, \kappa\rangle_{\mathsf{sk}} &\to \langle S_1, \Sigma, \lceil \texttt{let}\ p = \square\ \texttt{in}\ S_2, \Sigma\rfloor :: \kappa\rangle_{\mathsf{sk}} \\
\cdots &\to \cdots \\
\langle \lceil \texttt{let}\ p = \square\ \texttt{in}\ S, \Sigma\rfloor :: \kappa, r\rangle_{\mathsf{kr}} &\to \langle p, r, \Sigma, \lceil S, \square\rfloor :: \kappa\rangle_{\mathsf{pat}} \\
\langle \lceil S, \square\rfloor :: \kappa, \Sigma\rangle_{\mathsf{ke}} &\to \langle S, \Sigma, \kappa\rangle_{\mathsf{sk}}
\end{aligned}$$

Problem: still non-deterministic, so not really computable...
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Deterministic Abstract Machine

$$\begin{aligned}
\langle \mathtt{Branch}(S :: l), \Sigma, \kappa, f \rangle_{\mathsf{sk}} &\to \langle S, \Sigma, \kappa, \lceil\!\lceil \mathtt{Branch}(l), \Sigma, \kappa \rfloor\!\rfloor :: f \rangle_{\mathsf{sk}} \\
\langle \mathtt{Branch}([]), \Sigma, \kappa, f \rangle_{\mathsf{sk}} &\to \langle f \rangle_{\mathsf{fk}} \\
\langle \mathtt{let}\ \ p = S_1\ \ \mathtt{in}\ \ S_2, \Sigma, \kappa, f \rangle_{\mathsf{sk}} &\to \langle S_1, \Sigma, \lceil \mathtt{let}\ \ p = \square\ \ \mathtt{in}\ \ S_2, \Sigma \rfloor :: \kappa, f \rangle_{\mathsf{sk}} \\
\cdots &\to \cdots \\
\langle \lceil \mathtt{let}\ \ p = \square\ \ \mathtt{in}\ \ S, \Sigma \rfloor :: \kappa, r, f \rangle_{\mathsf{kr}} &\to \langle p, r, \Sigma, \lceil S, \square \rfloor :: \kappa, f \rangle_{\mathsf{pat}} \\
\langle \lceil S, \square \rfloor :: \kappa, \Sigma, f \rangle_{\mathsf{ke}} &\to \langle S, \Sigma, \kappa, f \rangle_{\mathsf{sk}} \\
\cdots &\to \cdots \\
\langle \lceil\!\lceil S, \Sigma, \kappa \rfloor\!\rfloor :: f \rangle_{\mathsf{fk}} &\to \langle S, \Sigma, \kappa, f \rangle_{\mathsf{sk}}
\end{aligned}$$

AM ⇒ NDAM