Aspects of a machine-checked intermediate language for extraction from Coq, in MetaCoq

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Extraction in Coq

Coq's Extraction turns 18 this year!

One of the central claims to fame of Coq

works by first erasing types and proofs, obtaining a term in the untyped lambda-calculus λ□

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Soutenue le 9 juillet 2004 devant la commission d'examen composée de

M.	LEROY Xavier	président
M.	BERARDI Stefano	rapporteurs
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Mme	BENZAKEN Véronique	examinateurs
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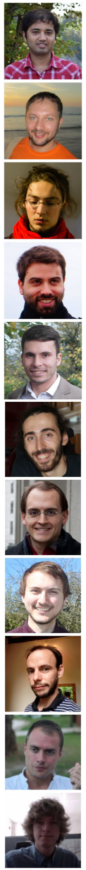
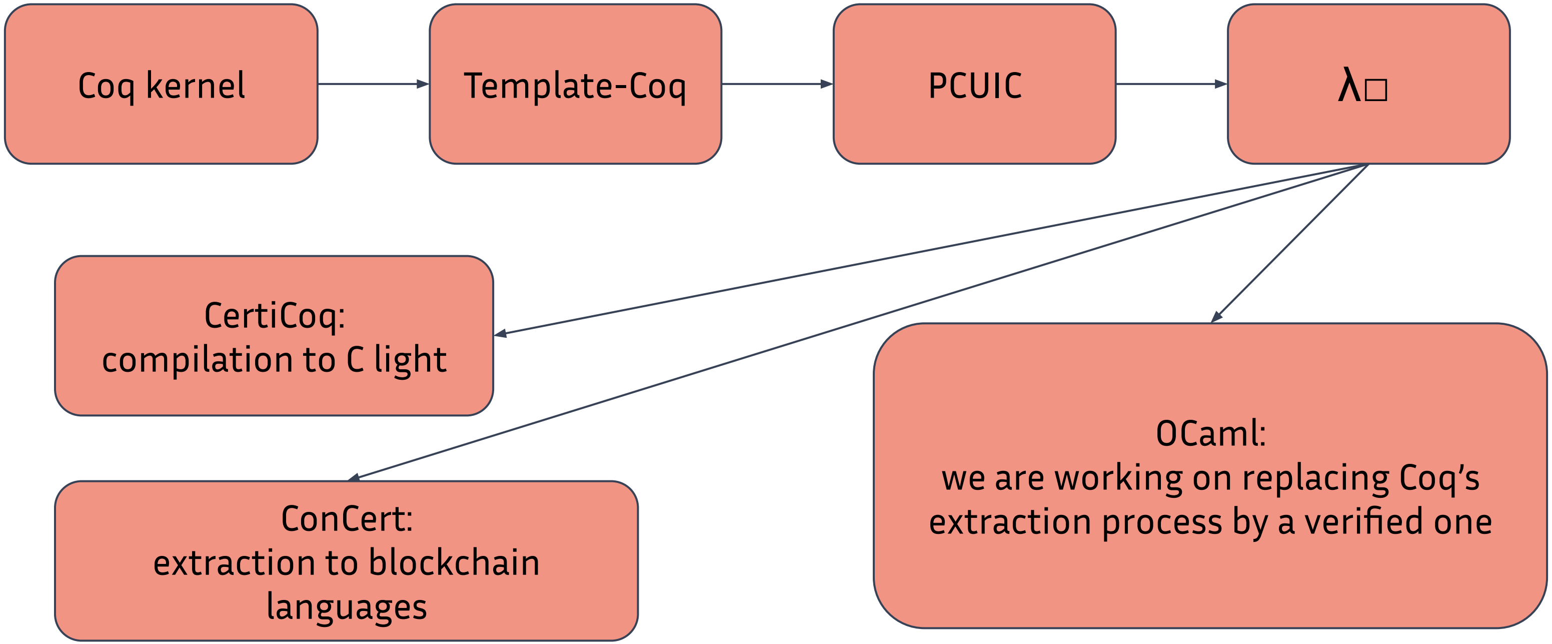
The MetaCoq project

- a formalisation of Coq in Coq
  - confluence, validity, subject reduction
  - weak call-by-value standardisation (if t is of first-order inductive type and reduces to a value, then this value can be found with weak call-by-value evaluation)
- machine-checked programs regarding Coq:
  - a correct and complete type checker
  - an erasure procedure into an untyped version of Coq, removing proofs
- Vision: a fast kernel for daily use, a verified kernel for monthly use
- Future work:
  - eta, talk to Meven Lennon-Bertrand
  - SProp, talk to Yann Leray
  - modules, talk to Yee Jian Tan
  - template polymorphism



Towards verified extraction



Subtle differences

Coq / lambda box

- structural fix
- higher-order constructors
- match on □
- de Bruijn
- fix / match

OCaml / C representation

- unary let rec
- constructors are blocks
- cannot match on functions
- named
- let rec / switch / proj

uniform solutions necessary,

to avoid doing work twice for different targets

Flags

```
(** Fix unfolding, with guard *)
| eval_fix f mfix idx argsv a av fn res :
    forall guarded : with_guarded_fix,
    eval f (mkApps (tFix mfix idx) argsv) ->
    eval a av ->
    cunfold_fix mfix idx = Some (#|argsv|, fn) ->
    eval (tApp (mkApps fn argsv) av) res ->
    eval (tApp f a) res

(** Fix stuck, with guard *)
| eval_fix_value f mfix idx argsv a av narg fn :
    forall guarded : with_guarded_fix,
    eval f (mkApps (tFix mfix idx) argsv) ->
    eval a av ->
    cunfold_fix mfix idx = Some (narg, fn) ->
    (#|argsv| < narg) ->
    eval (tApp f a) (tApp (mkApps (tFix mfix idx) argsv) av)

(** Fix unfolding, without guard *)
| eval_fix' f mfix idx a av fn res narg :
  forall unguarded : with_guarded_fix = false,
  eval f (tFix mfix idx) ->
  cunfold_fix mfix idx = Some (narg, fn) ->
  eval a av ->
  eval (tApp fn av) res ->
  eval (tApp f a) res
```

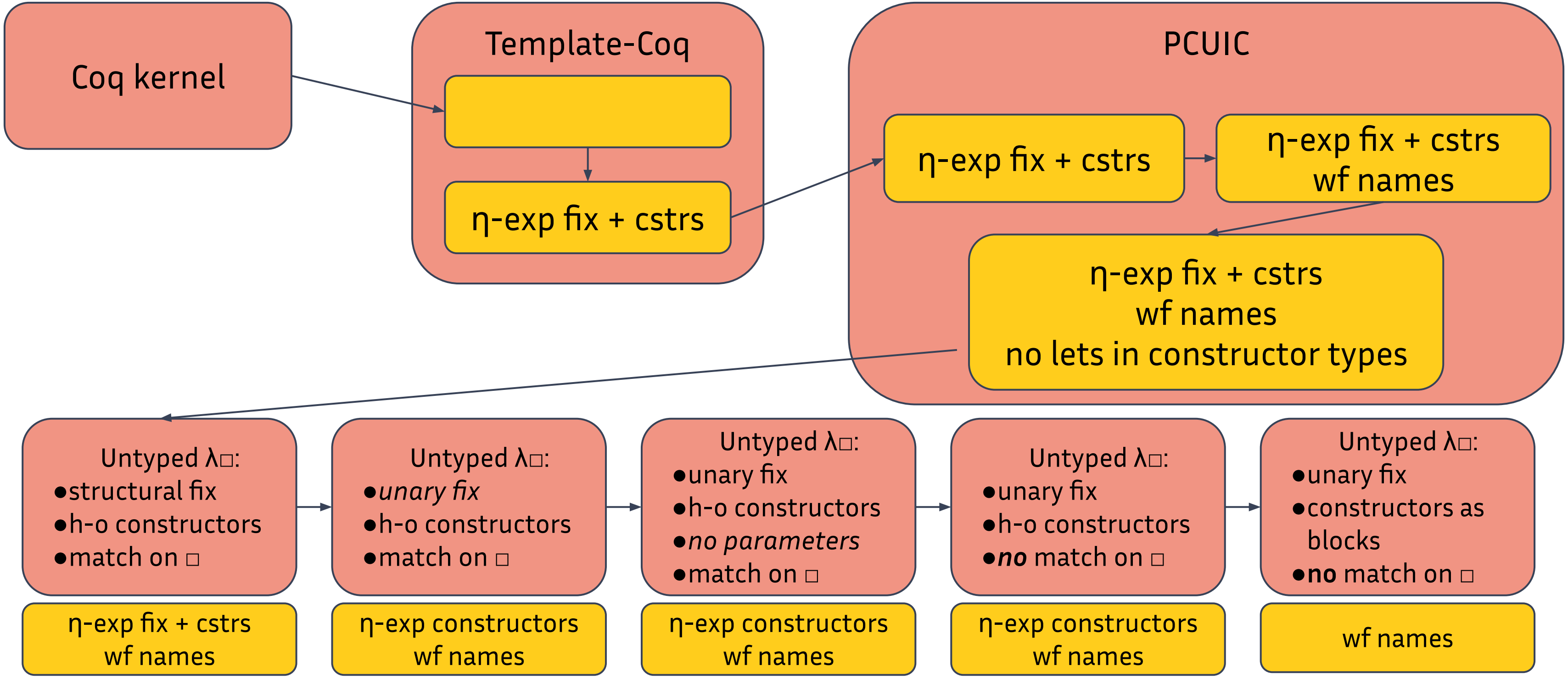
Economical proofs needed

1. “This theorem is true.”
2. “There is a proof of this theorem.”
3. “The proof of this theorem can be formalised.”
4. “The proof of this theorem can be formalised in less than a week.”

All problems are solvable by talking about observational congruence

All problems are easy to solve for terms where all constructors and fixpoints are eta-expanded

Both restructuring of proofs and proof engineering are central (missing UI features, missing nested induction principles: talk to Tomas Vallejos)


Coq kernel
Template-Coq
η-exp fix + cstrs
PCUIC
η-exp fix + cstrs
η-exp fix + cstrs
wf names
η-exp fix + cstrs
wf names
no lets in constructor types
Untyped λ□:
•structural fix
•h-o constructors
•match on □
η-exp fix + cstrs
wf names
Untyped λ□:
•*unary fix*
•h-o constructors
•match on □
η-exp constructors
wf names
Untyped λ□:
•unary fix
•h-o constructors
•*no parameters*
•match on □
η-exp constructors
wf names
Untyped λ□:
•unary fix
•h-o constructors
•***no*** match on □
η-exp constructors
wf names
Untyped λ□:
•unary fix
•constructors as blocks
•**no** match on □
wf names

Towards Verified Extraction from Coq to OCaml

Give a verified implementation of extraction

- formalise Coq in Coq
- formalise (a variant of) OCaml
- **re-implement extraction**
- **verify it**

`https://metacoq.github.io`

Joint work with Matthieu Sozeau, Pierre Giraud, Pierre-Marie Pedrót, and Nicolas Tabareau

Merci !

https://metacoq.github.io

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